A Fast Scheme for Downsampling and Upsampling in the DCT Domain

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Abstract

Given a video frame or image in terms of its 8×8 block-DCT coefficients we wish to obtain a downsized or upsized (by factor of two) version of this frame also in terms of 8×8 block-DCT coefficients. We propose an algorithm for achieving this directly in the DCT domain which is computationally much faster, produces visually sharper images and gives significant improvements in PSNR (typically 4 dB better) compared to other compressed domain methods based on bilinear interpolation. The downsampling and upsampling schemes combined together preserve *all* the low-frequency DCT coefficients of the original image. This implies tremendous savings for coding the difference between the original (unsampled image) and its prediction (the upsampled image). This is desirable for many applications based on scalable encoding of video.

1 Introduction

Due to the advances in digital signal processing and digital networks more and more video data is available today in digital format. For economy of storage and transmission digital video is typically stored in compressed format. However for many applications one needs to produce in real-time a compressed bitstream containing the video at a different resolution than the original compressed bitstream. For example for browsing a remote video database it would be more economical to send low-resolution versions of the video clip to the user and depending on his or her interest progressively enhance the resolution. Resizing of video frames is also required when a viewer is tuned to two different TV channels with frames from one of the channels being displayed in a corner of the screen. Similar need arises when converting between different digital TV standards or to fit the incoming digital video onto the user's TV screen. Similarly for transmitting video over dual-priority networks we can transmit a low-resolution version of the video over high-priority channel and an enhancement layer over the low-priority channel.

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The straightforward approach of decompressing, carrying out the downsampling in spatial domain and then recompressing involves unnecessary work and is computationally too intensive to be feasible in real-time on currently available workstations. Hence recently there has been much work [1, 2, 3] in carrying out downsampling as well as other operations on video sequences directly in the compressed domain without requiring decompression and recompression.

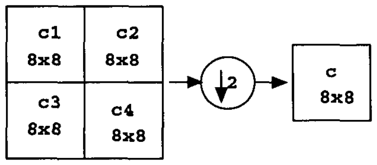

Figure 1: Downsampling 4 consecutive blocks to get a single block.

Most previous approaches for downsampling in the compressed domain (say in the DCT domain) rely on the fact that the DCT, being a linear orthonormal transform, is distributive over matrix multiplication [4, 1, 2]. Let c_1, c_2, c_3, c_4 denote four adjacent 8×8 blocks in the spatial domain as shown in Figure 1 then the downsampled 8×8 block c can be written as: $c = \sum_{i=1}^{4} h_i c_i g_i$ where the downsampling filters h_i, g_i are usually taken as

$$h_1 = \begin{bmatrix} .5 & .5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .5 & .5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & .5 & .5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & .5 & .5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (1)$$

and similarly for the other h_i's and g_i's. Let T denote the 8×8 DCT operator matrix. Since $TT^t = T^tT = I$

and $\mathrm{DCT}(c) = TcT^t$ we have

$$\mathrm{DCT}(c) = \sum_{i=1}^{4} \mathrm{DCT}(h_i)\mathrm{DCT}(c_i)\mathrm{DCT}(g_i) \quad (2)$$

$\mathrm{DCT}(h_i)$ and $\mathrm{DCT}(g_i)$ for i= 1 to 4 can be precomputed. Hence $\mathrm{DCT}(c)$ can be computed as matrix multiplication. However even though the filter matrices h_i and g_i are sparse, $\mathrm{DCT}(h_i)$ and $\mathrm{DCT}(g_i)$ are not sparse at all. Hence the matrix multiplication in (2) can have complexity comparable to downsampling in spatial domain (by first taking inverse DCT, filtering and taking DCT) using fast algorithms for computation of the DCT. In [1] a fast algorithm has been developed for the computation of (2) based on factorization of the DCT matrix corresponding to the Winograd algorithm. However their approach is mainly aimed at the downsampling matrix in Eq. (1) and it has been commented that the method is not guaranteed to have lesser complexity compared to spatial domain methods for every reasonable anti-aliasing filter.

Hence we see that in the above approaches the low-pass filter is chosen independent of the DCT transform. In our approach we shall show that it is possible to design a low-pass filter so that the DCT of the filter matrix is sparse rather than the filter matrix being sparse.

We shall first give an outline of our scheme for 1-D signals. Let $\mathbf{B}_1$ and $\mathbf{B}_2$ denote the 8-point DCT of two consecutive 8-sample blocks $\mathbf{b}_1$ and $\mathbf{b}_2$. We wish to generate the 8-point DCT $\mathbf{B}$ of the 8-point block got by downsampling ($\mathbf{b}_1$, $\mathbf{b}_2$) (by factor of 2) with an appropriate filter. The downsampling has to carried out as far as possible in the compressed domain. In principle the proposed scheme can be viewed as follows: Take 4-point inverse DCT of the 4 lowpass coefficients in $\mathbf{B}_1$ and similarly for $\mathbf{B}_2$. Concatenate these two 4-point blocks and then take its 8-point DCT. The resulting block is the desired block $\mathbf{B}$. In section 2 we shall describe an algorithm for implementing this operation and show that it is computationally faster compared to (2). The scheme works because taking 4 point inverse DCT of the 4 lowpass coefficients of 8-point DCT of a block gives a low passed and downsampled version of the original 8-point block [5].

2 Downsampling in the DCT domain

We shall derive the relevant equations in 1-D and then extend them to 2-D. As in the previous section let $\mathbf{b}_1$ and $\mathbf{b}_2$ denote two consecutive 8-pixel blocks in the spatial domain. Let $\mathbf{B}_1$ and $\mathbf{B}_2$ be their 8-point DCTs respectively. Let $\mathbf{b} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix}$. Let $\hat{\mathbf{B}}_1$ and $\hat{\mathbf{B}}_2$ denote the first 4 (low-pass) components of $\mathbf{B}_1$ and $\mathbf{B}_2$. Let $\hat{\mathbf{b}}_1$ and $\hat{\mathbf{b}}_2$ denote the 4-point inverse DCT of $\hat{\mathbf{B}}_1$ and $\hat{\mathbf{B}}_2$. Hence $\hat{\mathbf{b}}_1$ and $\hat{\mathbf{b}}_2$ are low-passed and downsampled versions of $\mathbf{b}_1$ and $\mathbf{b}_2$ respectively. Let $\hat{\mathbf{b}} = \begin{bmatrix} \hat{\mathbf{b}}_1 \\ \hat{\mathbf{b}}_2 \end{bmatrix}$ and let $\hat{\mathbf{B}}$ be the 8-point DCT of $\hat{\mathbf{b}}$. Hence $\hat{\mathbf{b}}$ is a low-pass filtered downsampled version of $\mathbf{b}$. We need to compute $\hat{\mathbf{B}}$ directly from $\mathbf{B}_1$ and $\mathbf{B}_2$ (i.e. from $\hat{\mathbf{B}}_1$ and $\hat{\mathbf{B}}_2$). Let T (8×8) denote the 8-point DCT operator matrix and let T_4 (4×4) denote the 4-point DCT operator matrix. Then we have the following equation:

$$\begin{aligned}\hat{\mathbf{B}} = T\hat{\mathbf{b}} &= T\begin{bmatrix} \hat{\mathbf{b}}_1 \\ \hat{\mathbf{b}}_2 \end{bmatrix} = [T_L \;\; T_R]\begin{bmatrix} T_4^t\hat{\mathbf{B}}_1 \\ T_4^t\hat{\mathbf{B}}_2 \end{bmatrix} \\ &= T_L T_4^t \hat{\mathbf{B}}_1 + T_R T_4^t \hat{\mathbf{B}}_2 \qquad (4)\end{aligned}$$

where T_L and T_R are 8×4 matrices denoting the first and last four columns respectively of the 8-point DCT operator T . Let us have a look at $T_L T_4^t$ and $T_R T_4^t$ and see how about 50% of the terms in the product turn out to be zeros (see also Eq. (3) at top): [1]

$$T_4 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ \cos\frac{\pi}{8} & \cos\frac{3\pi}{8} & -\cos\frac{3\pi}{8} & -\cos\frac{\pi}{8} \\ \cos\frac{2\pi}{8} & \cos\frac{6\pi}{8} & \cos\frac{6\pi}{8} & \cos\frac{2\pi}{8} \\ \cos\frac{3\pi}{8} & \cos\frac{9\pi}{8} & -\cos\frac{9\pi}{8} & -\cos\frac{3\pi}{8} \end{pmatrix} \quad (5)$$

The following points can be noted immediately from Eqs. (3) and (5) :

1. For $k = 0,1,2,3$, the $(2k)$th row of T_L is same as the kth row of T_4 and the the $(2k)$th row of T_R is negative of the kth row of T_4 .
2. Since the rows of T_4 are orthogonal, point 1 above implies that *every* $(2k)$th row of T_L (and also of T_R) is orthogonal to *every* row of T_4 except the kth row for $k = 0,1,2,3$. Hence in the 8×4 matrices $T_L T_4^t$ and $T_R T_4^t$ every $(2k)$th row has all its entries as zeros except the kth entry. Hence we see that about 50% of the entries of these matrices are zero.
3. Odd rows of T are anti-symmetric and even rows are symmetric. Also odd rows of T_4 are anti-symmetric and even rows are symmetric. These two facts together imply that all the corresponding entries of $T_L T_4^t$ and $T_R T_4^t$ are identical except possibly for a change of sign. Specifically $T_L T_4^t(i,j) = (-1)^{i+j} T_R T_4^t(i,j)$ for $i = 0,1,\ldots,7$ and $j = 0,1,2,3$. This fact can be exploited to further reduce the computations in Eq. (4). Grouping the terms for which $i + j$ is even into one matrix C and the remaining terms into another matrix D we have $T_L T_4^t = C + D$ and $T_R T_4^t = C - D$.

[1]The normalizing constants in these matrices have been omitted since they do not affect our conclusions.

$$T = [T_L \mid T_R] = \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \cos\frac{\pi}{16} & \cos\frac{3\pi}{16} & \cos\frac{5\pi}{16} & \cos\frac{7\pi}{16} & -\cos\frac{7\pi}{16} & -\cos\frac{5\pi}{16} & -\cos\frac{3\pi}{16} & -\cos\frac{\pi}{16} \\ \cos\frac{\pi}{8} & \cos\frac{3\pi}{8} & -\cos\frac{3\pi}{8} & -\cos\frac{\pi}{8} & -\cos\frac{\pi}{8} & -\cos\frac{3\pi}{8} & \cos\frac{3\pi}{8} & \cos\frac{\pi}{8} \\ \vdots & & & & & & & \vdots \end{array} \right) \quad (3)$$

Hence from Eq. (4) we have:

$$\begin{aligned} \mathbf{B} &= (C+D)\hat{\mathbf{B}}_1 + (C-D)\hat{\mathbf{B}}_2 && (6) \\ &= C(\hat{\mathbf{B}}_1 + \hat{\mathbf{B}}_2) + D(\hat{\mathbf{B}}_1 - \hat{\mathbf{B}}_2) && (7) \end{aligned}$$

Eq. (7) is computationally much faster compared to Eq. (6) because each of the matrices C and D have only half the number of non-zero entries compared to $C+D$ or $C-D$. Hence the number of multiplications involved is reduced drastically. Another way to look at this is that instead of computing expressions like $\alpha\beta + \alpha\gamma$ in Eq. (6) we are computing $\alpha(\beta+\gamma)$ in Eq. (7).

4. Since T and T_4 are DCT matrices it is our belief that much faster procedures (based on fast DCT algorithms [6]) could be designed for the scheme (cf. Eq. (4)) presented here. But we shall not pursue this in the current paper.

Using the facts mentioned above it can be shown that for the 2-D case our downsampling scheme requires 1.25 multiplications and 1.25 additions per pixel of the original image.

2.1 The Downsampling Filter

In this section we shall derive the downsampling filter which corresponds to the downsampling operation mentioned above. We have the following equation:

$$\hat{\mathbf{b}}_1 = T_4^t\hat{\mathbf{B}}_1 = T_4^t[I \quad O]\mathbf{B}_1 = T_4^t[I \quad O]T\mathbf{b}_1 = T_4^t M^t T\mathbf{b}_1 \quad (8)$$

where I and O denote 4×4 identity and zero matrices respectively and $M \stackrel{\text{def}}{=} \begin{bmatrix} I \\ O \end{bmatrix}$. Hence we have

$$\begin{bmatrix} \hat{\mathbf{b}}_1 \\ \mathbf{0} \end{bmatrix} = MT_4^t M^t T\mathbf{b}_1 \stackrel{\text{def}}{=} h\mathbf{b}_1 \quad (9)$$

where $\mathbf{0}$ is a 4×1 zero vector. Hence we see that the downsampling filter matrix $h(8\times8)$ is given by

$$h = MT_4^t M^t T \quad (10)$$

$$\text{DCT}(h) = ThT^t = [T_L \quad T_R]MT_4^t M^t TT^t = T_L T_4^t M^t \quad (11)$$

We have already seen above that $T_L T_4^t$ is very sparse. The M^t at the end makes sure that only the first four low-pass components of DCT($\mathbf{b}_1$) are used in the computation of $\hat{\mathbf{b}}_1$. Hence we have designed a filter matrix h such that its DCT (and not h) is sparse[2].

[2]Note that $\text{DCT}(h\mathbf{b}_1) = Th\mathbf{b}_1 = ThT^tT\mathbf{b}_1 = \text{DCT}(h)\text{DCT}(\mathbf{b}_1)$.

2.2 Extension to 2-D

Let $\mathbf{b}_1$, $\mathbf{b}_2$, $\mathbf{b}_3$ and $\mathbf{b}_4$ denote four consecutive 8×8 blocks numbered in the same way as c_1, c_2, c_3 and c_4 in Fig. 1. Following the same notation as in 1-D case, let $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$ and $\mathbf{B}_4$ denote the DCTs of these blocks respectively. Let $\hat{\mathbf{B}}_1$ etc. denote the 4×4 matrices containing the low-pass coefficients of $\mathbf{B}_1$ etc. Let $\hat{\mathbf{b}}_1$ etc. denote the 4×4 inverse DCT of $\hat{\mathbf{B}}_1$ etc. Then $\hat{\mathbf{b}} \stackrel{\text{def}}{=} \begin{bmatrix} \hat{\mathbf{b}}_1 & \hat{\mathbf{b}}_2 \\ \hat{\mathbf{b}}_3 & \hat{\mathbf{b}}_4 \end{bmatrix}$ denotes the low-pass and downsampled version of $\mathbf{b} \stackrel{\text{def}}{=} \begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 \\ \mathbf{b}_3 & \mathbf{b}_4 \end{bmatrix}$. Let $\hat{\mathbf{B}} \stackrel{\text{def}}{=} \text{DCT}(\hat{\mathbf{b}})$. We need to compute $\hat{\mathbf{B}}$ directly from $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$ and $\mathbf{B}_4$ (i.e. from $\hat{\mathbf{B}}_1$, $\hat{\mathbf{B}}_2$, $\hat{\mathbf{B}}_3$ and $\hat{\mathbf{B}}_4$). We have

$$\begin{aligned} \hat{\mathbf{B}} &= T\hat{\mathbf{b}}T^t && (12) \\ &= [T_L \; T_R] \begin{bmatrix} \hat{\mathbf{b}}_1 & \hat{\mathbf{b}}_2 \\ \hat{\mathbf{b}}_3 & \hat{\mathbf{b}}_4 \end{bmatrix} \begin{bmatrix} T_L \\ T_R \end{bmatrix} && (13) \\ &= [T_L \; T_R] \begin{bmatrix} T_4^t\hat{\mathbf{B}}_1 T_4 & T_4^t\hat{\mathbf{B}}_2 T_4 \\ T_4^t\hat{\mathbf{B}}_3 T_4 & T_4^t\hat{\mathbf{B}}_4 T_4 \end{bmatrix} \begin{bmatrix} T_L \\ T_R \end{bmatrix} && (14) \\ &= (T_L T_4^t)\hat{\mathbf{B}}_1 (T_L T_4^t)^t + (T_L T_4^t)\hat{\mathbf{B}}_2 (T_R T_4^t)^t \\ &\quad + (T_R T_4^t)\hat{\mathbf{B}}_3 (T_L T_4^t)^t + (T_R T_4^t)\hat{\mathbf{B}}_4 (T_R T_4^t)^t && (15) \end{aligned}$$

We have already seen that $(T_L T_4^t)$ and $(T_R T_4^t)$ are very sparse. We know from 1-D case that $(T_L T_4^t) = C+D$ and $(T_R T_4^t) = C-D$ where each of C and D contain only half as many non-zero entries as $(T_L T_4^t)$ or $(T_R T_4^t)$. Hence it can be shown that

$$\hat{\mathbf{B}} = (X+Y)C^t + (X-Y)D^t \quad (16)$$

where

$$\begin{aligned} X &= C(\hat{\mathbf{B}}_1 + \hat{\mathbf{B}}_3) + D(\hat{\mathbf{B}}_1 - \hat{\mathbf{B}}_3) && (17) \\ Y &= C(\hat{\mathbf{B}}_2 + \hat{\mathbf{B}}_4) + D(\hat{\mathbf{B}}_2 - \hat{\mathbf{B}}_4) && (18) \end{aligned}$$

We know from the 1-D case that Eqs. (17) and (18) are computationally inexpensive. Again from 1-D case we know that Eq. (16) is also computationally inexpensive. It can be shown that our downsampling scheme requires **1.25 multiplications and 1.25 additions** per pixel of the original image. Compare this to 4 multiplications and 4.75 additions (after assuming that the DCT of any 8×8 image block is 75% sparse) per pixel of original image required by the method of Chang *et. al* [2].

3 Upsampling

In many applications it is required to obtain an upsampled version from a downsampled version of an original image. For example for spatially scalable encoding of video [7, 8] a prediction of the original frame is obtained by upsampling the lower resolution frame and the difference is encoded in the enhancement or low-priority layer. A natural question is can we design an upsampling scheme that works in the compressed domain and can keep all the low-frequency components of the original image in the upsampled image. In terms of our notation in the previous section the problem is: can we get back $\hat{\mathbf{B}}_1$, $\hat{\mathbf{B}}_2$, $\hat{\mathbf{B}}_3$ and $\hat{\mathbf{B}}_4$ from $\hat{\mathbf{B}}$ (see Eq. (15)). Since the matrices T and T_4 are unitary the following inverse relationships can be easily derived from Eq. (15): [3]

$$\hat{\mathbf{B}}_1 = (T_L T_4^t)^t \hat{\mathbf{B}} (T_L T_4^t) \quad (19)$$
$$\hat{\mathbf{B}}_2 = (T_L T_4^t)^t \hat{\mathbf{B}} (T_R T_4^t) \quad (20)$$
$$\hat{\mathbf{B}}_3 = (T_R T_4^t)^t \hat{\mathbf{B}} (T_L T_4^t) \quad (21)$$
$$\hat{\mathbf{B}}_4 = (T_R T_4^t)^t \hat{\mathbf{B}} (T_R T_4^t) \quad (22)$$

Then

$$\tilde{\mathbf{B}}_1 \overset{\text{def}}{=} \begin{bmatrix} \hat{\mathbf{B}}_1 & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} \quad (23)$$

etc. give us the four blocks in the upsampled image corresponding to the block $\hat{\mathbf{B}}$ in the downsampled image. Note that the process of downsampling and then upsampling has the effect of truncating to zero all the high frequency components in each 8×8 DCT block in the original image while preserving all the low-frequency components of such blocks. Also note the upsampling scheme can be applied to any given image irrespective of whether or not the given image is obtained by the downsampling scheme outlined in Section 2. It can be shown that our upsampling scheme requires 1.25 multiplications and 1.50 additions per pixel of the upsampled image.

4 Results

We have already shown the computational efficiency of our method to be superior to existing methods for downsampling in the DCT domain. Moreover, since we preserve most of the significant energy (the low-pass coefficients of each block) of the image, the image obtained after downsampling and upsampling by our method looks much sharper and preserves more texture compared to the image obtained after bilinear interpolation. The later looks blurred. The table below gives the PSNR values when an image is downsampled and upsampled using our method and the bilinear interpolation method. The Cap image is obtained from

Image	Lena	Watch	F-16	Cap
our	34.69	29.09	32.28	34.22
bilinear	30.04	25.15	28.11	32.08

ftp://ipl.rpi.edu/pub/image/still/KodakImages/color/03.ras.
Comparing the PSNR shows that the image obtained by our downsampling and upsampling scheme is typically about 4 dB better than the image obtained by bilinear interpolation. Figure 2 makes a visual comparison. Detailed results can be found at *http://vision.ai.uiuc.edu/~dugad/draft/icip99dct.html.*

[3]Actually due to the different sizes of the DCTs there would be a factor of half that we have not accounted for so far. Hence in practice Eqs. (17)-(22) would be modified as: $X = \frac{1}{2}[C(\hat{\mathbf{B}}_1 + \hat{\mathbf{B}}_3) + D(\hat{\mathbf{B}}_1 - \hat{\mathbf{B}}_3)]$ (and similarly for Y) and $\hat{\mathbf{B}}_1 = 2(T_L T_4^t)^t \hat{\mathbf{B}} (T_L T_4^t)$ etc.

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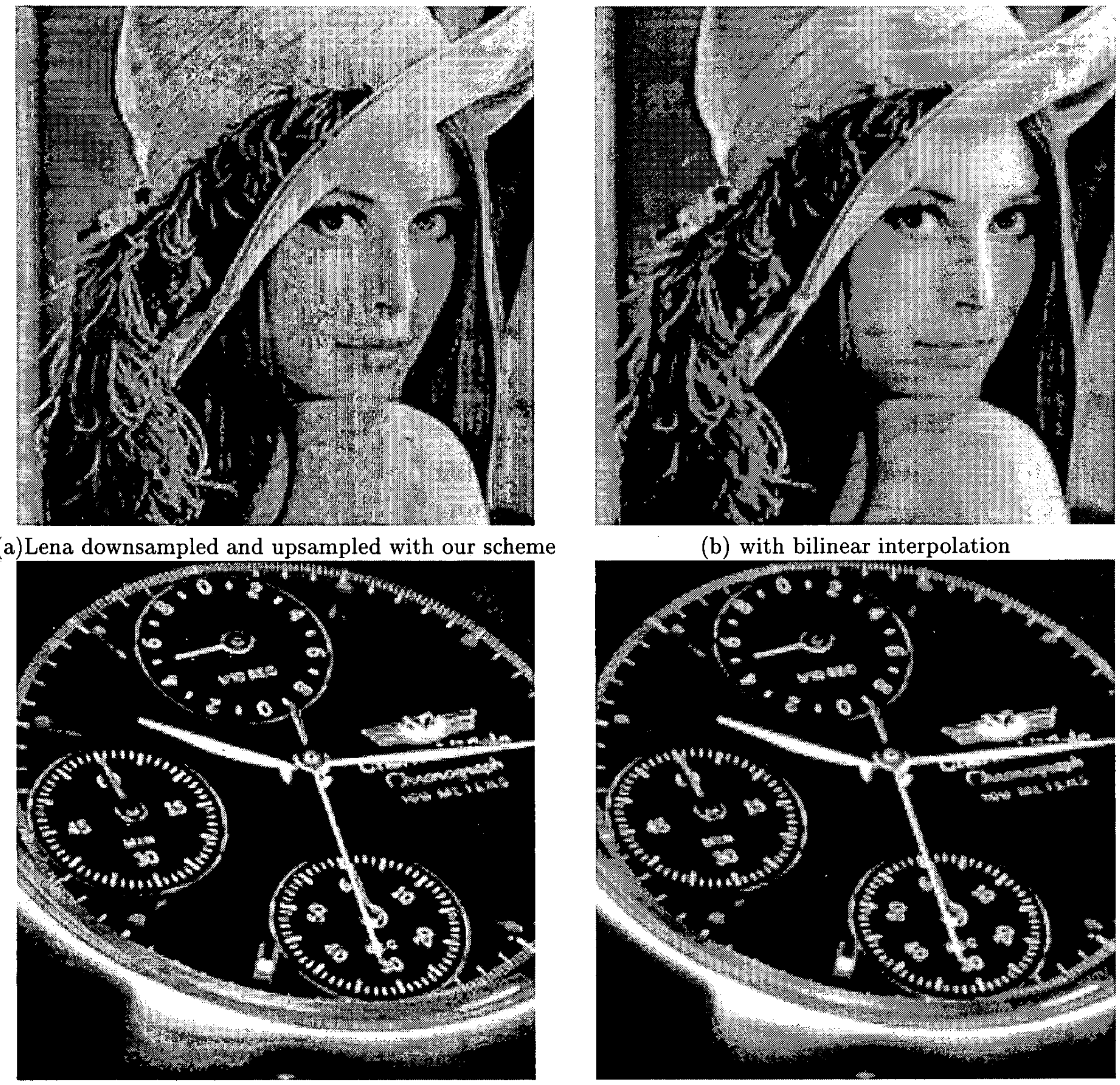

(a)Lena downsampled and upsampled with our scheme

(b) with bilinear interpolation

(c) Watch downsampled and upsampled with our scheme

(d) with bilinear interpolation

Figure 2: Notice the sharpness of the Lena image in (a) compared to (b) especially in the hair, eyes and cap region. For the Watch image the numbers on the three smaller dials are much clearer in (c) compared to (d).