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PATH PLANNING USING THE NEWTONIAN POTENTIAL

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Abstract

Potential functions are used to represent the topological structure of free space in solving path planning problems because of the simplicity in the free space representation and the guidance provided by the negative gradient of the potential field in the form of repulsive force. In this paper, Newtonian potential function is used to represent polygonal objects and obstacles. The closed-form expression of this potential field as well as some other gradient-related quantities are derived. Such results not only eliminate the problems associated with the discretization of the object and obstacles in evaluating the risk of collision but also make the search for the optimal object configurations efficient. The object skeleton, a shape description of the moving object, is introduced to guide the moving object through narrow regions while the above search is done at different stages. The free space can then be divided by the narrow regions where the above path planning takes place — a very simple free space decomposition scheme. Successful global strategies are developed to connect the above local plans into a safe and smooth global path.

1. Introduction

The problem of path planning is determining how to move an object from its original location and orientation (called the starting configuration) to the goal configuration while avoiding collisions with obstacles. The objects and obstacles may be of different kinds: they may be rigid or deformable, they may be moving or stationary, and the motion of the objects may be constrained or unconstrained. This paper considers path planning for a single rigid object among stationary, rigid obstacles. Although the approach presented applies to three-dimensions and to complex shapes, we will confine the detailed discussion to two-dimensional (2D) polygonal objects and obstacles for concreteness.

1.1. Previous Work

Algorithms to solve the path planning problems usually consist of two parts. In the first part, relevant information about the free space is extracted, which is then used in the second part to destination. This second part is executed in two major ways, giving rise to different kinds of algorithms. Algorithms of the first type use known obstacle space as reference and plan a path barely avoiding the obstacles, e.g., moving object while keeping in contact with the obstacles. A solution is guaranteed if there is one. The configuration approach [1] and the critical curve approach [9] are examples of this type of algorithms. These algorithms are useful in solving hard path planning problems and are more complicated than those of the second type.

Algorithms of the second type consider the shape of free space to plan a path through the space. The free space is represented in different ways by different algorithms. Octree representation of the three-dimensional free space is used in [3]. Rectangular corridors and their junctions are used in [8] to represent the free space among rectangular obstacles. Convex areas are used in [10] for path planning of a point object. [7] uses generalized cylinders and convex polygons to represent free space. In [11], circular discs are used to create the generalized Voronoi diagram (GVD) defined by obstacle location and shapes. The Voronoi edges are then used to derive paths for rectangular objects. Because of the simplifications in the free space representation, these algorithms are faster than those of the first type but their applications are limited to easy path planning problems where tight maneuvering is not necessary.

Both of the above types of algorithms do not have any basis for choosing the object configurations to match obstacle shapes while generating a path. Such a flexibility is especially useful for the second type of algorithms wherein the object orientations are to be identified along the path so as to avoid collision with obstacles. One way of measuring the risk of collision and thus choosing a minimum risk orientation is to define a repulsive potential field for the obstacles. Such a potential field can itself serve as a representation of free space. [12] uses a potential function which is a cubic function of the distance between a point object and the obstacles. [5] uses an artificial repulsive potential, which is the function of the shortest distance between the moving object and the obstacles, for local planning of linked line segments. Similar local planning is done in [6] using a superquadric artificial potential function whose isopotential contours are modified n-ellipses and the potential values are determined by the Yukawa function [2]. Boundary equations of polytopes are used in [4] to create an artificial potential function. An initial path is optimized by minimizing a cost function which depends on the repulsive potential and the object motion. The main advantages of such potential field approaches include the simplicity of the representation of free space, the guidance in the object motion provided by the negative gradient of the potential field in the form of repulsive force, and the readiness of its extension to spaces of higher dimensions.

1.2. Motivation and Approach

The approach presented in this paper decomposes the path planning problem into several subproblems. The decomposition is defined by the bottlenecks in the free space around which the risk of collision is very high. Maneuvering through each such tight part of free space is considered as a separate local subproblem. Traversal of free space between the bottlenecks is used to define the second set of subproblems. The solutions of the second set of subproblems provide the links between the solutions of the first set, thus yielding a global solution. For the 2D problems considered in this paper, bottlenecks are defined by the minimal distance links (MDL's) among polygonal regions, which can be computed easily.

The risk of collision between the object to be moved and the obstacles is measured through the Newtonian potential model where each object/obstacle region border is assumed to be charged. Near an MDL, shape matching between the object and the free space is important and the proposed approach uses object skeleton to guide the object motion. Local strategy moves the object through each MDL such that successive skeleton points cross it sequentially. The exact location and orientation of the object as the successive skeleton points enter the bottleneck are determined by the force and torque experienced by the object. Thus, the object moves in a way so as to reduce the experienced force and torque to zero. The local solution terminated once the last skeleton point exits the MDL. It is shown in Section 2 that repulsive force and torque between polygonal regions due to such a model are analytically tractable.

Global planning is used to link the solutions of the local problems at adjacent bottlenecks into a global solution. Once the object exits a bottleneck, it is *pulled* by the next MDL by forcing the object to reduce its distance to the MDL. As the object is being pulled, it is allowed to change its location and orientation so as to minimize the force and torque experienced. Thus, the object is forced to follow the minimal risk path and configuration between bottlenecks and because the distance to the next MDL decreases monotonically, a solution is guaranteed to be found if one exists. In this sense, the local minima of potential fields do not present a problem.

For brevity, in this paper it is assumed that the sequence of MDL's to be traveled through by the object is given, as is the sequence of the object skeleton points to cross each MDL.

2. Using the Newtonian Potential

Artificial potential fields formed by obstacles have been used previously to represent the topological structure of the free space. Either the potential function or its negative gradient, the repulsive force, can be used to plan paths that avoid obstacles. Ideally, such potential fields should have the following attributes, which are similar to those mentioned in [6]:

1. The magnitude of potential should be unbounded near the obstacle boundary and should decrease with range. (Property captures the basic requirement of collision avoidance.)
2. The potential should have spherical symmetry far away from the obstacle.
3. The equipotential surfaces near the obstacle should have shape similar to the obstacle surface.
4. The potential, its gradient and their effects on derived paths must be spatially continuous.

The Newtonian potential is known to satisfy these properties. The approach presented in this paper ensures that the resulting potential field possesses the following additional properties:

5. The potential and its gradient experienced at a point due to a simple obstacle primitive can be calculated with respect to any subset of obstacle points, and only object points are included in the calculation.
6. The potential, and repulsive force and torque between primitives other than points are analytically tractable.

None of the previously discussed artificial potential fields satisfy either Property 5 or 6. Efficient computation of the gradient, force and torque is central to the approach described in this paper. This is accomplished by assuming that the obstacles and object are polygonal, as described in the following subsections.

2.1. Newtonian Potential/Force from Line Segments

For the 2D case shown in Figure 1, consider a point A at $(0, y_0 \neq 0)$ and a finite line charge on x-axis with a unit charge density uniformly distributed between $x=x_1$ and $x=x_2$. The Newtonian potential at point A due to the whole line segment can be calculated as

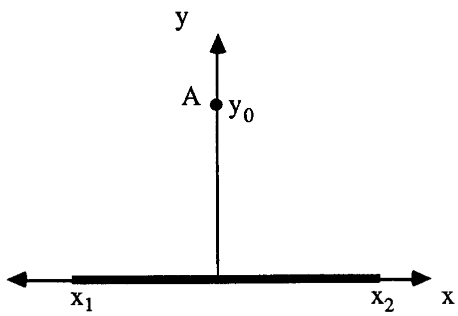


Figure 1. A finite line charge and a point A in a selected coordinate system.

$$\phi_A = \int_{x_1}^{x_2} \frac{dx}{r} = \int_{x_1}^{x_2} \frac{dx}{\sqrt{x^2+y_0^2}} = \log \frac{|x_2+\sqrt{x_2^2+y_0^2}\,|}{|x_1+\sqrt{x_1^2+y_0^2}\,|}. \quad (1)$$

If there is more than one line segment, the total potential is simply the sum of the potential values due to each individual line segment according to the superposition principle.

If $y_0 = 0$, point A and the line charge are collinear and

$$\phi_A = \int_{x_1}^{x_2} \frac{dx}{\sqrt{x^2}} = \begin{cases} \log|x_2/x_1| & \text{if } x_1 > 0, \\ \log|x_1/x_2| & \text{if } x_2 < 0, \end{cases} \quad (2)$$

which is the limiting case of (1) when $x_1 \cdot x_2 > 0$. When $x_1 \cdot x_2 \leq 0$, it can be shown from (1) that,

$$\lim_{\substack{y_0 \to 0 \\ x_1 \cdot x_2 \leq 0}} \phi_A = \infty. \quad (3)$$

The value of ∞ is desirable for the contact detection and collision avoidance in the path planning since $y_0 = 0$ and $x_1 \cdot x_2 \leq 0$ corresponds to a collision between point A and the line charge.

The negative gradient of the potential function is the repulsive force experienced by a point charge of unit strength. Moving along its direction corresponds to a maximal reduction rate of the potential away from that point. The repulsive force on point A in Figure 1 due to a point $(x, 0)$ on the x-axis is

$$-\nabla\left[\frac{1}{r}\right] = \frac{1}{r^2}\hat{\mathbf{r}} = \frac{1}{x^2+y_0^2}(\cos\theta\hat{\mathbf{x}}+\sin\theta\hat{\mathbf{y}}),$$

where θ is the angle of the position vector with respect to the $+x$ direction. Therefore, the resulting force from the line segment on A has the following two components:

$$F_x = \int_{x_1}^{x_2} \frac{\cos\theta\, dx}{r^2(x)} = \int_{x_1}^{x_2} \frac{-x\, dx}{(x^2+y_0^2)^{3/2}} = \frac{1}{\sqrt{x^2+y_0^2}}\Bigg|_{x_1}^{x_2} \quad (4)$$

and

$$F_y = \int_{x_1}^{x_2} \frac{\sin\theta\, dx}{r^2(x)} = \int_{x_1}^{x_2} \frac{y_0\, dx}{(x^2+y_0^2)^{3/2}} = \frac{x}{y_0\sqrt{x^2+y_0^2}}\Bigg|_{x_1}^{x_2}, \quad (5)$$

where F_x is the force along the x-axis and F_y is the force along the y-axis.

2.2. Repulsion between Line Segments

In the previous subsection, the potential and its gradient at an (object) point due to an (obstacle) line segment is derived in a form such that it is readily calculable from just the two end points of that line segment. Not only is the calculation time reduced dramatically, but the problems associated with sampling are also eliminated. Analogously, it is desirable to discuss the interaction of two line segments so it is possible to calculate the effect on the object line segment due to an obstacle line segment.

For the discussion in this subsection, only the repulsion between *nonparallel* line segments is considered. (Discussions for other simpler cases are omitted.) First, the force on a point of an (object) line segment due to repulsive forces from points of another (obstacle) line segment is derived as a function of that point's location on the first line segment. This result can then be used to derive the repulsive torque and force between line segments.

2.2.1. Force on a Point of a Line Segment

Consider two line segments, $\overline{ab}$ and $\overline{cd}$ shown in Figure 2, the coordinates of the end points of $\overline{cd}$ are assumed to be (0,0) and $(d_1 \geq 0, 0)$, possibly after a corresponding coordinate transformation. The repulsive force on point $(x, 0)$ due to line charge $\overline{ab}$, can be expressed as

$$\mathbf{F}(x) = F_{ab}(x)\hat{\mathbf{n}}_{ab} + F_{ab}^{+}(x)\hat{\mathbf{n}}_{ab}^{+}, \quad (6)$$

where $\hat{\mathbf{n}}_{ab}$ is the unit vector along the $\mathbf{b}-\mathbf{a}$ direction, $\hat{\mathbf{n}}_{ab}^{+}$ is a vector perpendicular to $\overline{ab}$, and, from (4) and (5),

$$F_{ab}(x) = \frac{1}{\sqrt{x^2+bx+c}} - \frac{1}{\sqrt{x^2+ex+f}} \quad (7)$$

and

$$F_{ab}^{+}(x) = \frac{1}{gx+h}\left[\frac{ix+j}{\sqrt{x^2+bx+c}} - \frac{kx+l}{\sqrt{x^2+ex+f}}\right], \quad (8)$$

where b, c, e, f, g, h, i, j, k, and l are constants since x_1, x_2, and y_0 are *linear* functions of x.

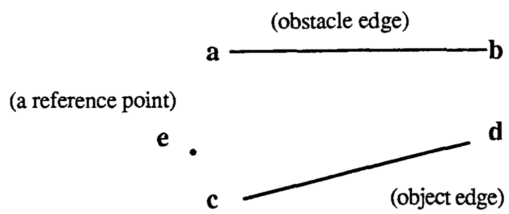


Figure 2. Two line segments and a reference point for the calculation of the repulsive force and torque due to the Newtonian potential.

2.2.2. Total Repulsive Torque and Force

Consider the simplest case of repulsion involving only two line segments, as shown in Figure 2. Let $\overline{ab}$ be the stationary obstacle, $\overline{cd}$ be the object and $\mathbf{e}$ be a fixed point. The torque with respect to $\mathbf{e}$, due to the repulsive force from $\overline{ab}$, on point $\mathbf{x} = (x, 0)$ of $\overline{cd}$ is

$$\mathbf{T}(x) = \mathbf{F}(x) \times (\mathbf{x} - \mathbf{e}), \tag{9}$$

and is in either $+z$ or $-z$ direction. From (6) through (9), we have

$$T_z(x) = \frac{gx+a}{\sqrt{x^2+bx+c}} - \frac{gx+a}{\sqrt{x^2+ex+f}} + \frac{(ix+d)(ix+j)}{(gx+h)\sqrt{x^2+bx+c}} - \frac{(kx+d)(kx+l)}{(gx+h)\sqrt{x^2+ex+f}}, \tag{10}$$

where a and d are constants. The total torque due to repulsion between $\overline{ab}$ and $\overline{cd}$ is then

$$T_z = \int_0^{d_1} T_z(x)dx. \tag{11}$$

Since $\overline{ab}$ and $\overline{cd}$ are not parallel, $g \neq 0$ and (11) can be simplified as

$$T_z = \frac{1}{g}\left[\int_0^{d_1} \frac{xdx}{\sqrt{x^2+bx+c}} - \int_0^{d_1} \frac{xdx}{\sqrt{x^2+ex+f}}\right] + \frac{1}{g^2}\left[\int_0^{d_1} \frac{ag^2+(di+ji)g-hi^2}{\sqrt{x^2+bx+c}}dx - \int_0^{d_1} \frac{ag^2+(di+li)g-hi^2}{\sqrt{x^2+ex+f}}dx\right] + \frac{dg-hi}{g^3}\left[\int_{h'}^{d_1+h'} \frac{(jg-hi)dx}{x\sqrt{x^2+b'x+c'}} - \int_{h'}^{d_1+h'} \frac{(lg-hi)dx}{x\sqrt{x^2+e'x+f'}}\right], \tag{12}$$

where $h' = h/g$, b', c', e', and f' are constants.

Similarly, the total repulsive force between the two line segments shown in Figure 2 along an arbitrary direction $\mathbf{e}$ can be expressed as

$$F_e = \left[d - \frac{ai}{g}\right]\left[\int_0^{d_1} \frac{dx}{\sqrt{x^2+bx+c}} - \int_0^{d_1} \frac{dx}{\sqrt{x^2+ex+f}}\right] - \frac{a}{g^2}\left[\int_{h'}^{d_1+h'} \frac{(jg-hi)dx}{x\sqrt{x^2+b'x+c'}} - \int_{h'}^{d_1+h'} \frac{(lg-hi)dx}{x\sqrt{x^2+e'x+f'}}\right]. \tag{13}$$

(12) and (13) can be evaluated analytically using an integral table and are used in the path planning strategies discussed in the following sections to generate object configurations of minimal risk of collision.

3. Local Planning

In this section, a local strategy is developed for path planning around an MDL where collision is more likely to occur and shape matching between the object and the free space is important. Object skeleton is used to guide the motion around the MDL such that the given sequence of skeleton points cross the MDL sequentially. The connection problem caused by using a single reference point of the object (see Figure 3) can thus be avoided. A simple and natural strategy is developed to allow for maximal freedom in adjusting the object configuration for minimal risk of collision using (12) and (13).

Consider the L-shaped object shown in Figure 4(a). The sequence of points on its skeleton to be used for motion through the MDL is given. The local planning

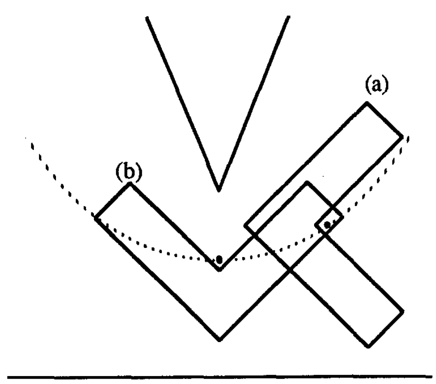
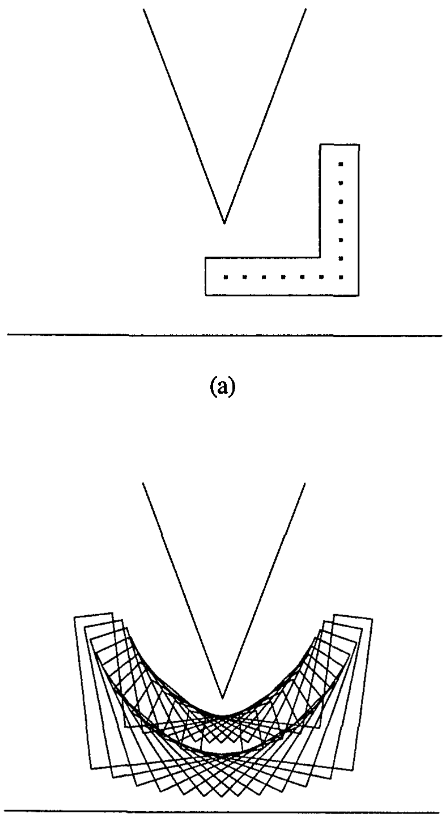


Figure 3. Object configurations (a) and (b) can not be related by a translation followed by a rotation without any contact with the obstacles.



(a)

(b)

Fig. 4. Path planning by shifting the object skeleton points to the MDL and finding the minimal Newtonian potential object configuration for each point constrained to lie on the MDL: (a) initial conditions, (b) result.

starts when the first skeleton point to cross the MDL (the *pilot* point) reaches the MDL, as shown in Figure 4(a), and ends when the last one leaves the MDL. The local planning is performed by ensuring that each skeleton point stays on the MDL while the object may adjust its orientation to achieve the minimal risk (minimal Newtonian potential) object configuration.

In the implementation, the above minimal risk configuration is obtained using a univariant search for object location and orientation along directions provided by the repulsive force and torque, respectively. The accuracies required in specifying the final object location and orientation determine the number of iterations needed in solving the corresponding constrained optimization problem. The local path thus obtained is safe and smooth, as shown in Figure 4(b), because the potential function is spatially smooth (in the configurational space) and maximal freedom is allowed in the adjustment of the object configuration for acquiring minimal potential. Figure 5 shows another local planning example.

4. Global Path Planning from Local Plans

A global strategy is described in this section to connect two neighboring local plans by moving the object from one MDL toward another so that the initial condition for the latter, similar to that shown in Figure 4(a), is established. The pilot point is pulled by the next MDL. The pulling effect is achieved by reducing the perimeter of the triangle determined by the next MDL and the pilot point. For each fixed perimeter value, the location of the pilot point is constrained to lie on an ellipse while the Newtonian potential is minimized (by letting the object reorient and relocate). In contrast to using a line to constrained a sequence of skeleton points in the local planning algorithm, multiple ellipses are used to constrain a single skeleton point - the pilot point.

Figure 6 (a) shows a problem containing two MDL's. Given the orders in which skeleton points should cross these links (same order in this case), the local plans are connected into a safe and smooth global path with the above global strategy, as shown in Figure 6(b). The step sizes along the planned path are determined adaptively during the planning process to be as large as possible as long as no collision between the object and obstacles is detected between neighboring stages. The simulation takes about 20 seconds for each local plans and one minute to connect them on a Sun 3/260 Workstation.

5. Summary and Future Research

In this paper, path planning using the Newtonian potential representation is studied. This is accomplished with the combination of the following procedures:

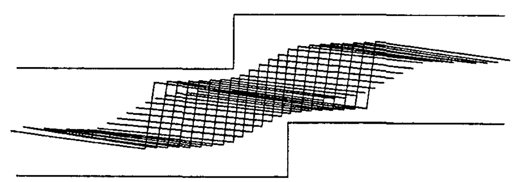
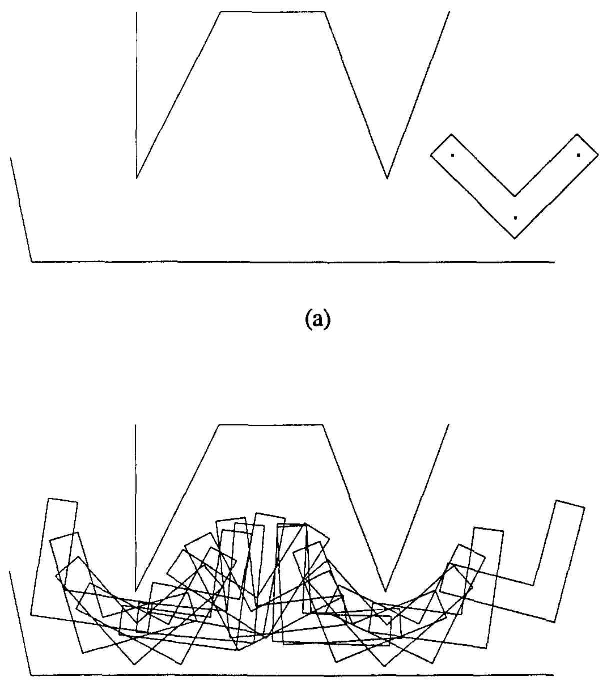


Figure 5. Another local planning example.



(a)

(b)

Figure 6. Path planning by connecting local plans using a global strategy: (a) initial conditions, (b) result.

1. Decompose the free space by narrow regions corresponding to bottlenecks.
2. Move the object through each narrow region using a local strategy which make the object skeleton traverse the MDL while minimizing a Newtonian potential based measure of risk of collision.
3. Connect the local plans using a global strategy.

The above algorithm has been implemented and tested on synthetic data. The performance appears to be satisfactory with respect to collision avoidance, smooth object motion and the execution speed.

More research is needed for the development of a systematic way of obtaining the sequence of bottlenecks to be traveled through by the object and the sequence of the skeleton points to cross each of them. Other research directions include the extension of the proposed approach to the 3D space and to more complex objects, e.g. objects with flexible joints.

References

[1] R. A. Brooks and T. Lozano-Perez, "A Subdivision Algorithm in Configuration Space for Findpath with Rotation," *Int. Joint Conf. on Artificial Intelligence,* Karlsruhe, Germany 1983.

[2] B. Cohen-Tannoudji, C. Diu, and F. Laloe, *Quantum Mechanics,* Volume 2, John Wiley and Sons, New York, 1977.

[3] M. Herman, "Fast, Three-Dimensional, Collision-Free Motion Planning," *Proc. of IEEE Int. Conf. on Robotics and Automation,* San Francisco, California, April 1986.

[4] Y. K. Hwang and N. Ahuja, "Path Planning Using a Potential Field Representation," Coordinated Science Laboratory, University of Illinois, UILU-ENG-88-2251, October 1988.

[5] O. Khatib, "Real-Time Obstacle Avoidance for Manipulators and Mobile Robots," *Proc. of IEEE Int. Conf. on Robotics and Automation,* St. Louis, March 1985.

[6] P. Khosla and R. Volpe, "Superquadric Artificial Potentials for Obstacle Avoidance and Approach," *Proceedings of IEEE Int. Conf. on Robotics and Automat.,* Philadelphia, Pennsylvania, April 1988.

[7] D. T. Kuan, J. C. Zamiska, and R. A. Brooks, "Natural Decomposition of Free Space for Path Planning," *Proceedings of IEEE Int. Conf. on Robotics and Automat.,* St. Louis, Missouri, March 1985.

[8] S. R. Maddila, "Decomposition Algorithm for Moving a Ladder Among Rectangular Obstacles," *Proc. of IEEE Int. Conf. on Robotics and Automation,* San Francisco, California, April 1986.

[9] J. T. Schwartz and M. Sharir, "On the Piano Movers' Problem: I. The Case of a Two-Dimensional Rigid Polygonal Body Moving Amidst Polygonal Barriers," *Communications on Pure and Applied Mathematics,* vol. 36, pp. 345-398, 1983.

[10] S. Singh and M. D. Wagh, "Robot Path Planning Using Intersecting Convex Shapes," *Proc. of IEEE Int. Conf. on Robotics and Automation,* San Francisco, California, April 1986.

[11] O. Takahashi and R. J. Schilling, "Motion Planning in a Plane Using Generalized Voronoi Diagrams," *IEEE Trans. Robotics Automat.,* vol. 5, no. 2, pp. 143-150, 1989.

[12] C. E. Thorpe, "Path Planning for a Mobile Robot," *Proceedings of AAAI,* Austin, Texas, 1984.