ISOTROPIC ERROR DIFFUSION HALFTONING

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ABSTRACT

The inherently causal nature of conventional single-pass error diffusion (ED) halftoning results in asymmetric diffusion of error. This results in the introduction of directional artifacts in the output halftone. In this paper we propose a novel two-pass algorithm which achieves symmetric error diffusion by using a zero-phase signal transfer function. We determine conditions under which isotropic diffusion of error and noise suppression are achieved. Experimental results demonstrate that the proposed algorithm breaks up worms and randomizes their direction, thus making the output halftone more visually appealing as compared to conventional error diffusion.

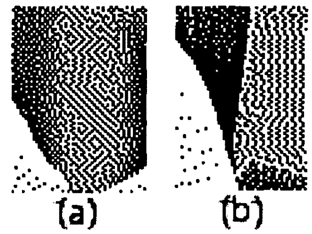

Fig. 1. Cropped parts of halftoned *lena* at 96 × 96 dpi: (a) depicts diagonal worms, (b) depicts vertical worms.

1. INTRODUCTION

Digital halftoning is the process of representing a continuous tone image by a perceptually truthful bilevel image. Among the various digital halftoning algorithms available in literature, the error diffusion algorithm (originally proposed in [1]) is one of the most popular ones due to the high perceptual quality of generated halftones. Figure 2 shows the block diagram for error diffusion. Let $x(i,j)$ denote the $(i,j)^{th}$ pixel of the input continuous tone image. In error diffusion, the current input gray level pixel $x'(i,j)$ is quantized to a one bit value $b(i,j) = Q(x'(i,j))$. The quantization error $e(i,j) = x'(i,j) - b(i,j)$ is weighed and *causally* 'diffused' to future pixels $x(i,j)$. The distribution of the error $e(i,j)$ is based on the error diffusion filter $h(m,n)$.

As shown in Figure 1, conventional one-pass error diffusion (ED) leads to the introduction of directional artifacts in the halftone. This has been referred to in halftoning literature as the introduction of worms. Work on sigma-delta modulation refers to this phenomenon as the introduction of limit cycles or idle tones. Kite et al. [2] argue that these refer to the same phenomenon and are due to the asymmetric distribution of error[1]. They also argue that isotropic diffusion of error would lead to these artifacts being less noticeable. However, isotropic error diffusion is not possible in single-pass ED owing to the causal nature of the ED process.

Previous work on suppressing these artifacts using single-pass ED includes the use of random perturbation (dithering) of the error filter weights or quantization thresholds to break up directional artifacts (for example, the void-and-cluster dither method [4, 13]). This however, results in the addition of perceptual noise to the halftone. Jarvis [5] and Stucki [6] propose the use of error filters with larger support to reduce these artifacts. Other algorithms use non-standard scans (eg. the serpentine scan [7]) to achieve this goal. The above approaches are constrained by the inherent causality of single-pass diffusion and are thus unable to achieve isotropic error diffusion.

Iterative schemes have been proposed in [8, 9] to achieve symmetric, non-causal error diffusion (or equivalently zero-phase diffusion of error). However, iterative schemes typically have high computational complexity. For example, the scheme proposed in [8] is reported to typically require about 10 iterations for convergence. Fan [10] proposes a two-pass error diffusion algorithm to achieve symmetric error diffusion. However, his algorithm diffuses error symmetrically only in the horizontal direction - anticausal feedback is absent in the vertical direction, and thus the diffusion of error is not truly with zero phase.

We note that achieving isotropic diffusion of error without incurring the computational complexity of iterative techniques is a difficult problem. This can be seen as follows: while halftoning pixel $x'(i,j)$, the error is diffused to its adjoining pixels. To achieve isotropic diffusion of error, this error needs to be diffused in all directions. However the error cannot be diffused to pixels halftoned prior to $x(i,j)$ since they have already been quantized. In this work, we alleviate this problem by halftoning an input 256 gray level image to an intermediate number of levels (say 16) using causal ED and then halftoning this intermediate image to a bilevel image anti-causally. The proposed algorithm ensures isotropic diffusion of error by choosing the intermediate level such that the zero-phase condition is explicitly satisfied. We determine conditions under which isotropic error diffusion is achieved and select algorithm parameters to further ensure noise suppression. Results demonstrate that the proposed algorithm reduces directional artifacts without incurring the high computational costs of iterative techniques [8],[9].

The paper is organized as follows: Section 2 briefly reviews the model of ED used in the paper. Section 3 describes the proposed algorithm. Results are presented in Section 4. A summary of the algorithm and the results is given Section 5.

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[1] Knox [3] showed that this problem (which he refers to as the 'knight's move' problem, see Figure 3) is caused by the causal nature of the error filter.

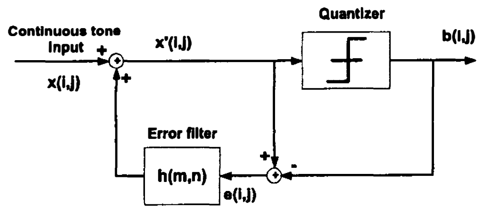

Fig. 2. Error Diffusion Block Diagram

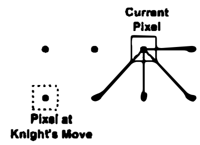

Fig. 3. The pixel at the knight's move does not receive error feedback (even indirectly) from the pixel being halftoned

2. MODELING ERROR DIFFUSION

We use the model proposed in [2, 11] in our work. In this section we briefly describe this model.

Figure 4 shows the equivalent circuit of error diffusion with the quantizer replaced by a constant linear gain K and independent additive noise $q(i,j)$. The system transfer function is as follows:

$$B(z) = \frac{K}{1+(K-1)H(z)}X(z) + (1-H(z))Q(z) \quad (1)$$
$$= S(z)X(z) + N(z)Q(z) \quad (2)$$

where $S(z)$ and $N(z)$ are the signal transfer function (STF) and the noise transfer function (NTF) respectively. $z = [z_x, z_y]^t$ denotes the two-dimensional z-transform.

The linear gain K in the model is an empirically determined quantity that models the correlation between the error image and the original image. It is noted in [2], that the value of K depends on the choice of the filter $H(z)$ and is image-independent. For the Floyd-Steinberg [1] filter, [2] estimates $K \approx 2$.

We note that in order to achieve isotropic diffusion of error, the STF $S(z)$ should have zero phase. In the case of single-pass error diffusion, this is unachievable since the error filter $H(z)$ is causal and thus has non-zero phase. In the next section, we show how a zero-phase STF can be achieved using a two-pass algorithm.

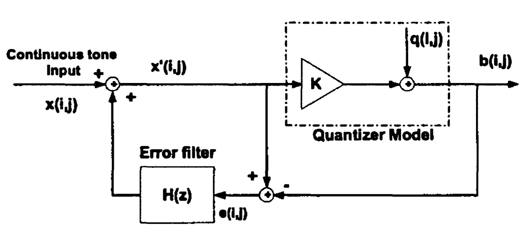

Fig. 4. Equivalent circuit for error diffusion with quantizer modeled as linear gain plus additive noise

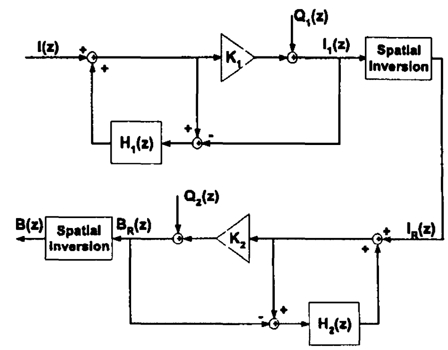

Fig. 5. Equivalent circuit of proposed algorithm

3. PROPOSED ALGORITHM

The proposed algorithm uses two error diffusion passes to achieve a zero phase STF. In the first pass (forward pass), the input image is halftoned, using a causal filter, so as to reduce the number of gray levels to an intermediate number of levels $\Delta_{in} > 2$. This intermediate image is then halftoned in the opposite direction (backward pass) using a *anti-causal* filter to obtain the bi-level output image. As we shall see later, choosing Δ_{in} judiciously leads to isotropic diffusion of error. The algorithm can be formally described as follows:

1. In the first pass, the input continuous graytone image $I(i,j)$ is quantized to a uniform intermediate quantization level $\Delta_{in} > 2$. The conventional error diffusion procedure is used except that the quantizer produces a multilevel output instead of a bilevel output.
2. The output image $I_1(i,j)$ from the first pass is spatially inverted along the vertical and horizontal directions resulting in an image $I_R(i,j) = I_1(n-i, m-j)$ where n, m are the horizontal and vertical dimensions of I.
3. In the second pass, error diffusion is used to quantize $I_R(i,j)$ to a bilevel output $B_R(i,j)$.
4. $B_R(i,j)$ is spatially inverted resulting in halftoned image $B(i,j)$.

3.1. Conditions for Isotropic Error Diffusion

In this section we determine conditions for isotropic distribution of error. Figure 5 shows the equivalent circuit model of the proposed algorithm, using the gain plus additive noise quantization model. $K_1$and K_2 are the equivalent linear gains of the two passes and $H_1(z)$, $H_2(z)$ are the causal FIR error filters used in the two passes. Then:

$$I_1(z) = S_1(z)I(z) + N_1(z)Q_1(z) \quad (3)$$
$$I_R(z) = I_1(z^{-1}) \quad (4)$$
$$B_R(z) = S_2(z)I_R(z) + N_2(z)Q_2(z) \quad (5)$$
$$B(z) = B_R(z^{-1}) \quad (6)$$

Here $S_i(z)$ and $N_i(z)$are the STF and NTF, respectively, for the ith pass.

Thus,

$$B(z) = S_2(z^{-1})S_1(z)I(z) + N(z) \quad (7)$$

where $N(z)$ is a noise term that is independent of the input image $I(z)$. The equivalent STF is therefore equal to the product $S_2(z^{-1}).S_1(z)$, where:

$$S_2(z^{-1}).S_1(z) = \frac{K_2(\Delta_{in})}{1+(K_2(\Delta_{in})-1)H_2(z^{-1})} \cdot \frac{K_1(\Delta_{in})}{1+(K_1(\Delta_{in})-1)H_1(z)} \quad (8)$$

where the functional dependence of the gain factors K_1, K_2 on the intermediate quantization level Δ_{in} has been emphasized. Since the step-size of the first quantizer is Δ_{in}, the value of K_1 is dependent on the intermediate quantization level Δ_{in}. Similarly, since the input to the second quantizer is itself coarsely quantized (with step size Δ_{in}) rather than continuous, the value of K_2 is also dependent on Δ_{in}.

For isotropic error diffusion, we require that this equivalent STF have zero phase. From (9), we note that this is achieved if the following relation holds:

$$H_2(z) = \left(\frac{\frac{K_2(\Delta_{in})}{K_1(\Delta_{in})}-1}{K_2(\Delta_{in})-1}\right) + \frac{(K_2(\Delta_{in}))(K_1(\Delta_{in})-1)}{(K_1(\Delta_{in}))(K_2(\Delta_{in})-1)}H_1(z) \quad (9)$$

(9) relates the filter $H_2(z)$ to K_1, K_2 and $H_1(z)$. Next, we select values for these parameters such that (9) is satisfied and the effect of the quantization noise is suppressed.

3.2. Parameter Selection for Noise Suppression

The equivalent NTF is:

$$\begin{aligned} N(z) &= S_2(z^{-1})N_1(z)Q_1(z) + .N_2(z^{-1})Q_2(z^{-1}) \quad (10) \\ &= S_2(z^{-1})(1-H_1(z))Q_1(z) + \quad (11) \\ &\quad (1-H_2(z^{-1}))Q_2(z^{-1}) \end{aligned}$$

As pointed out by Ulichney in [7], the addition of blue noise achieves visually pleasing halftones. Equivalently, the NTF $N(z)$ should be highpass. From (12), we see that this holds if both $(1-H_1(z))$ and $(1-H_2(z))$ are high pass filters (equivalently, if $H_1(z)$ and $H_2(z)$ are low-pass filters). Choosing $H_1(z)$ as a low-pass filter, K_1 and K_2 should be selected so as to ensure that $H_2(z)$ is also low-pass. Ensuring that $H_2(z)$ be lowpass is equivalent to ensuring that first term in (9) (which is allpass) vanishes. Thus, for $N(z)$ to be highpass, the conditions required are: (1) $H_1(z)$ should be lowpass (2) Δ_{in} should be selected such that

$$K_1(\Delta_{in}) = K_2(\Delta_{in}) \quad (12)$$

Further, from (9), we see that $K_1 = K_2$ results in $H_2(z) = H_1(z)$. Hence, the proposed algorithm achieves a zero-phase equivalent STF with noise suppression if the linear gains and error filters of the two passes are equal and the error filters are lowpass .

The functional dependence of the gain K on the quantizer step-size is unknown for the case of 2D error diffusion. However, as in [2], we observe that the gains K_1, K_2 are image-independent and depend on the choice of $H_1(z)$, $H_2(z)$. Then, for a given error filter, it is possible to estimate the value of Δ_{in} for which the zero-phase condition is satisfied, by using a set of test images to experimentally calculate the K_1, K_2 values for a range of Δ_{in} values. K_1, K_2 can be estimated by performing a least-squares fit of $e(i,j)$ and $x'(i,j)$. The experimentally determined value of Δ_{in} that satisfies (12) can then be used for halftoning any input images.

4. RESULTS

This section presents a comparison of the proposed algorithm with previous approaches that aim at removing directional artifacts. Results are presented for a 256-level gray-scale input *lena* image. All halftones are displayed at a resolution of 144 × 144 dpi. Results of [9] and [4] have been taken from [12].

For the proposed algorithm, the value of the intermediate number of grey levels Δ_{in} was determined as follows. For a given error filter, the values of $K_1$and K_2 were experimentally estimated over a range of intermediate number of grey levels . The value of Δ_{in} for which (12) was selected. As expected, the values of $K_1$and K_2 (and Δ_{in}) were found to be image-independent and were found to be dependent on the error filter used. Further, it was found that the value of K_1 is nearly unity for most values of Δ_{in}. This is in accordance with classical quantization theory which predicts that quantization error is independent of the input signal for high-resolution quantization. As Δ_{in} decreases, the value of K_1 was found to increase (since the first-pass quantizer became increasingly coarser) while K_2 decreased (as the second-pass quantizer became increasingly finer).

Figure 6(a) shows the halftone produced by conventional error diffusion using the Floyd Steinberg filter [1]; clusters of worms can be clearly seen under the right eye, around the nose and running down the entire left side of the face. Further, in each cluster, worms orient along a common direction. Figure 6(b) shows the halftone produced by the serpentine scan algorithm [7]. The serpentine scan reduces worms at 45^0, but does not break up the vertical worms running down the left side of the face.

Figure 6(c) shows the halftone produced by ordered dithering [4]. As can be seen, dithering breaks up worms, but at the expense of adding considerable perceptual noise. Figure 6(d) shows the halftone obtained by using the iterative algorithm proposed in [9]. The algorithm results in the addition of considerable noise to the image. It also causes loss of sharpness i.e. the structural details of the image are poorly reproduced. Moreover, the algorithm has high computational complexity; it requires 30 iterations to produce the final halftone.

Figure 6(e) shows the halftone produced by the proposed algorithm using the FS error filter for both passes. The value of Δ_{in}, selected as explained above, was 6 i.e. the intermediate image produced at the end of the first pass had 6 grey levels. In Figure 6(a) and Figure 6(b), worms cluster in predominantly one direction (such as the vertical worms on the left side of the face) resulting in easily observed artifacts. By contrast, worms are shorter and randomized in Figure 6(e), illustrating the isotropic diffusion of error produced by the proposed algorithm. Further, the halftone produced by the proposed algorithm is considerably less noisy as compared to Figure 6(c) and Figure 6(d).

Figure 6(f) shows the halftone produced by the proposed algorithm using the 3 × 5 error filter proposed in [8] H=[0 0 0 0.15 0.10; 0.06 0.10 0.15 0.10 0.06; 0.03 0.06 0.1 0.06 0.03]. $\Delta_{in} = 5$ was found to satisfy (12) and was used to generate results using the filter proposed in [8]. This error filter is designed such that the frequency response does not have a bandpass nature. As can be seen, the output halftone is considerably sharper than in the other cases. Also, directional artifacts are completely absent in the halftone. However the halftone is noisier as compared to Figure 6(a) and Figure 6(e). This illustrates an important feature of the proposed algorithm - any error filter can be used in the implementation and the remaining algorithm parameters can be accordingly determined. Thus, the error filter can be selected on the basis of the requirements on the output halftone. Specifically, the error filter selected results in a tradeoff between the addition of noise in smooth regions and faithful reproduction of high-frequency and textured regions.

5. SUMMARY

In this paper, we have proposed a two-pass algorithm that achieves symmetric error diffusion by explicitly ensuring a zero-phase STF. The key step is the choice of the intensity range (Δ_{in}) of the intermediate image. The proposed algorithm breaks up worms and randomizes their direction, thus making the output halftone more visually appealing as compared to conventional error diffusion. The proposed algorithm has the advantages of low-complexity, compared to iterative schemes, image-independence.

6. REFERENCES

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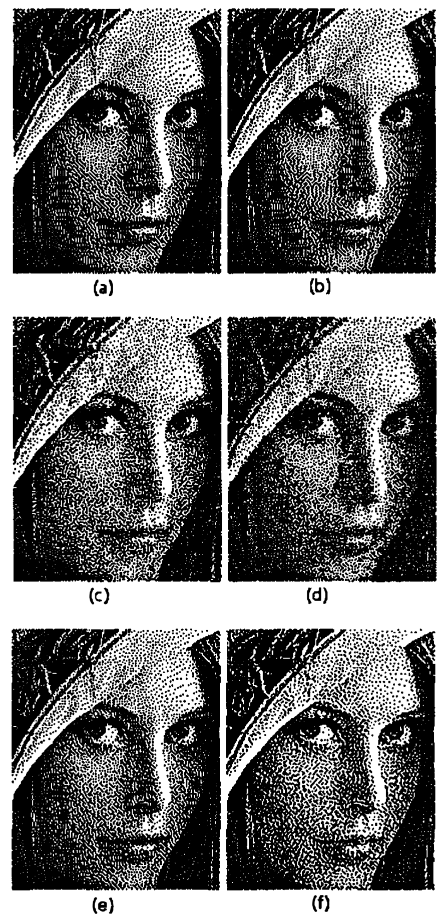

Fig. 6. *lena* halftoned using: (a) Floyd Steinberg Algorithm. (b) Floyd Steinberg with serpentine scan. (c) Void-and-cluster ordered dither. (d) Zeggel Bryngdahl's iterative algorithm. (e) Proposed algorithm using the Floyd Steinberg filter. (f) Proposed algorithm with the filter proposed in [8]. (Halftoned images are shown at 144×144 dpi).