Adaptive Polynomial Modelling of the Reflectance Map for Shape Estimation from Stereo and Shading *

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Abstract

This paper is concerned with estimation of the reflectance map and its use in surface shape estimation by a method that combines stereo and shading information. There are many advantages to the integrated approach. However, shape from shading algorithms are limited in their applicability by the assumption of idealized reflectance and lighting models involving many preset or hard to estimate parameters. In this paper, it is shown that the difficulties involved in estimation of the reflectance function and light source distribution can be eliminated through direct estimation of the reflectance map using adaptive polynomial models. The reflectance map estimate is used with a surface estimate provided by stereo in an integrated approach to surface shape estimation.

1 Introduction

This paper is concerned with estimation of the reflectance map and its use in surface shape estimation by a method that combines stereo and shading information. There are many advantages to combining stereo and shading information in an integrated approach [7, 4, 5]. However, shape from shading algorithms are limited in their applicability by the assumption of idealized reflectance and lighting models involving many preset or hard to estimate parameters.

Many approaches to shape from shading are based on the assumption that the surface reflectance function is given and that the light source distribution can be estimated [7, 4, 5, 10]. In addition, a method has recently been developed for using range images to estimate the parameters of a simple surface reflectance function [6].

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The key to the present approach is the fact that the reflectance map contains all of the information about the light source distribution and the surface reflectance function that is necessary and useful for recovering surface shape. Therefore, if the goal is to estimate surface shape, it is not necessary to estimate the surface reflectance function or light source distribution. It is sufficient to estimate the reflectance map.

The method of adaptive polynomial modelling is presented and its application to estimation of the reflectance map given an initial surface estimate is described. The algorithm for estimation of the final surface shape is a generalization of that presented by Bichsel and Pentland [1] which makes use of the initial depth map produced by stereo.

2 Adaptive Polynomial Modelling

Estimation of the reflectance map may be viewed as a multivariate regression problem with the reflectance map $R(n_1, n_2)$ an unknown function of the first two components of the surface normal. The boundedness of the normal components makes this a convenient parameterization for polynomial regression. A model for R containing a minimum number of parameters is desirable as it will be used in the computationally intensive shape from shading procedure. One method for solving this problem is the method of *adaptive polynomial regression*, suggested by Barron in [3].

Consider estimating the regression function $R(\boldsymbol{n}) = \mathcal{E}(E|\boldsymbol{n})$ where $\mathcal{E}$ is expectation and E is image irradiance. One possible model for R is a weighted sum of basis functions, $B_1, \ldots, B_M$ with corresponding weights, $\theta_1, \ldots, \theta_M$.

Adaptive regression is an automatic stepwise procedure in which basis functions are selected to minimize an appropriate criterion function, Q. The selected functions are combined to form a succession of models, $R_1, \ldots, R_M$. In the case of additive models, the Mth model is defined by $R_M(\boldsymbol{n}, \boldsymbol{\theta}) = \sum_{j=1}^{M} \theta_j B_j(\boldsymbol{n})$. The

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models are fit to the data, $(\boldsymbol{n}_i, E_i)_{i=1}^N$ and the model, R_M, obtaining the best fit with M terms becomes the Mth estimate, $\hat{R}_M$. In the case of d-variate polynomial regression, the class of models is the set of all d-variate polynomials of finite degree.

A strategy for searching the space of possible models is to consider the set of d-variate terms that can be constructed from terms already in the model. If the degree of any term is allowed to increase by a bounded amount, the set of basis functions considered at the $M+1$st step is $\Omega_M = \{n_1^{m_1} \cdots n_d^{m_d} B_j(\boldsymbol{n}) : 1 \leq m_1 + \cdots + m_d \leq M;\ j = 1, \ldots, M\} - \mathcal{A}_M$ where $\mathcal{A}_M = \{B_j(\boldsymbol{n}) : j = 1, \ldots, M-1\}$.

The basis function selected is that achieving the minimum value of a criterion function defined as the sum of the average squared error, $\mathrm{ASR} = \frac{1}{N}\sum_{i=1}^N (E_i - R(\boldsymbol{n}_i, \boldsymbol{\theta}))^2$ and a roughness penalty, $\mathrm{RP} = \frac{\delta^2}{N}\sum_{i=1}^N ||\nabla_{\boldsymbol{n}} R(\boldsymbol{n}_i, \boldsymbol{\theta})||^2$. This criterion provides an estimate of the average squared error that would be incurred if the inputs were perturbed slightly. Additional basis functions are included in the model until a minimum description length stopping criterion is reached; $\mathrm{MDL} = \mathrm{ASR} + \frac{M}{N}\sigma^2 \log(N)$.

3 Shape from shading algorithm

A computationally stable method for shape from shading is the optimal control approach developed by Dupuis and Oliensis [2] and simplified by Bichsel and Pentland [1] through the introduction of a 'minimum downhill principle'.

Consider moving towards the point (x, y) from a neighboring point in the direction $\pi + \phi$. The slope $\sigma(\phi)$ corresponds to the directional derivative of the surface in the direction ϕ and is given by $\sigma(\phi) = \cos(\phi)p + \sin(\phi)q$ where $(p, q) = (z_x, z_y)$. The slope in the direction ϕ must be chosen to fulfill the brightness equation, $E(x, y) = R(p, q)$. If the point (x, y) has brightness, E, then $(p, q) = (p(\alpha_E), q(\alpha_E))$ for some α_E where α_E parameterizes the level curve at level E of the reflectance map. As shown in [1], the choice of α_E which maximizes $\sigma(\phi) = \cos(\phi)p(\alpha_E) + \sin(\phi)q(\alpha_E)$ leads to the correct surface shape. The slope is always a positive distance from the peak of the reflectance map, ensuring algorithm convergence.

Applying the minimum downhill principle and propagating information from neighboring points towards (x, y) the update equation for the surface becomes,

$$z^{t+1}(x, y) = \sup_{\phi}[z^t(x + ds\cos(\phi), y + ds\sin(\phi)) - ds\,\sigma(\phi)]$$

at every point except those singular points that are specified in advance.

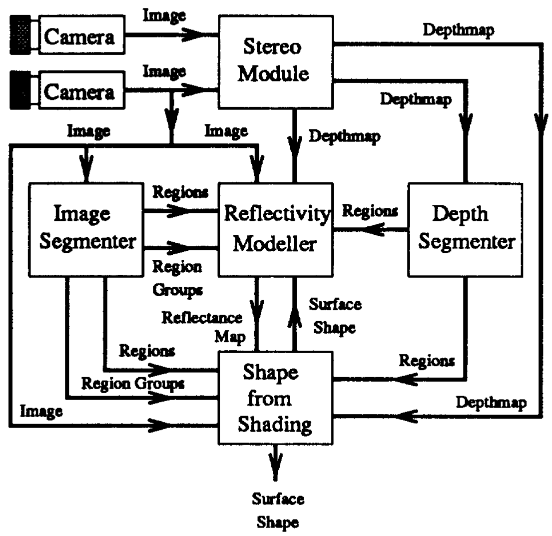

Figure 1: Data flow diagram.

In order to use the shading information in an integrated system, it is necessary to modify the update equation to include stereo information. To avoid complications related to the accuracy of stereo methods, an averaging approach is used here. The modified update equation is given by $\tilde{z}^{t+1}(x, y) = (1 - \gamma)z^{t+1}(x, y) + \gamma z_s(x, y)$ where z_s is the stereo depth map and γ is a small weight that controls the stereo contribution. The effect of this approach is to automatically select the highest singular points on the surface. The method works because the slope in the reflectance map near singular points is small and the stereo contribution dominates near these points. Conversely, where the slope in the reflectance map is large, the shading contribution dominates.

4 Other algorithms and integration

This section describes other modules of the complete approach and the method by which the various component algorithms are integrated. Figure 1 shows the data paths between the major blocks: (1) The stereo module operates on the input images and uses the known camera geometry to produce an initial estimate of the depth map that is passed to the depth segmenter, the reflectivity modeller and the shape from shading program. (2) The depth segmenter identifies depth discontinuities and divides the depth map into regions of roughly constant depth. This information is passed to the reflectivity modeller and shape from shading program for use in determining depth map reliability and, in the case of shape from shading, to assertain an overall depth constant for each region. (3) The image segmenter operates on one of the input images and produces a set of homogeneously colored regions and an associated set of region groups. Regions identified by the depth segmenter but not the image segmenter are considered erroneous depth estimates. Both depth and image segmentation are performed using the algorithm of Tabb and Ahuja [8]. (4) The reflectivity modeller uses the input image and the reliable parts of the initial depth map estimate to produce a reflectance map for each region group. (5) The shape from shading program generates an improved estimate of the surface shape in each image region using the reflectance map for each region group and the reliable depth values provided by the stereo module.

5 Results

The image in figure 2 is a gray scale rendition of a 500×500 color picture of of a child's toy puzzle with three-dimensional colored pieces. The object was placed approximately 1.0 meter from the camera and the pieces rise no more than 1.0 cm above the frame. Therefore, a relative error of much less than 1% must be achieved in order to accurately recover the surface shape.

The top right of figure 2 shows the result of image segmentation. The segmentation contains some errors that are partially responsible for errors in the final reconstruction. The lower left of figure 2 shows the stereo depth reliability. The regions that have been labelled unreliable by the algorithm are outlined.

The lower right of figure 2 shows the level curves of one reflectance map. From figures 3 through 5 it can be seen that the integrated approach results in a significant improvement in surface shape recovery over the unaided stereo algorithm or stereo aided by trimmed mean filtering.

The graphs in figures 3 and 5 show the depth maps and level curves produced by stereo, filtering the stereo output, and by the current approach.

The pictures in figure 4 show the results of stereo and filtered stereo illuminated by a source at $(1,1,1)/\sqrt(3)$ and the final results illuminated by sources at $(1,1,1)/\sqrt(3)$ and $(0,0,1)$.

It can be seen that very small features of the surface such as the musical score have been recovered and some larger features, such as the hands were accurately recovered. Other regions show the persistence of errors resulting from stereo and segmentation problems. However, it is clear that the result is much better than that achieved using only stereo information.

6 Conclusions

This paper has presented a direct method of calculating the reflectance map that avoids both the complexity of estimating the parameters of nonlinear surface reflectance functions and the complexity of estimating the light source distribution. The reflectance map is modelled as a polynomial function of the components of the normal vector. Results indicate that the integrated approach provides much more accurate results than those obtainable from stereo alone.

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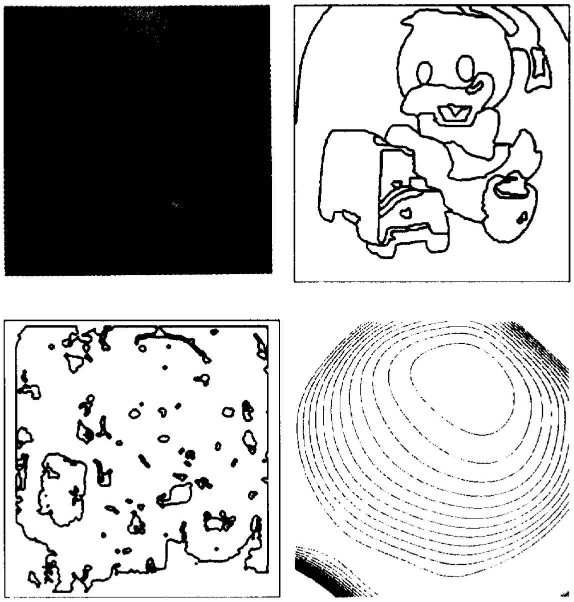

Figure 2: Image of Donald Duck, outlines of the image segmentation and depth segmentation, and level curves of a reflectance map.

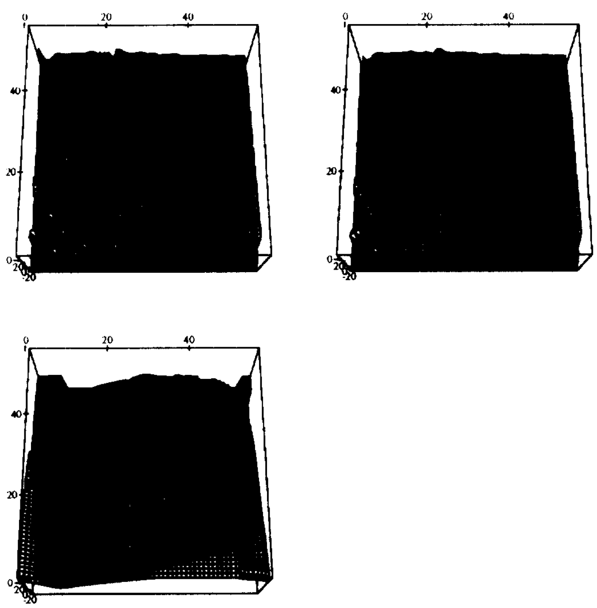

Figure 3: Depth map from stereo, filtered depth map from stereo, and final depth map produced by integrated approach.

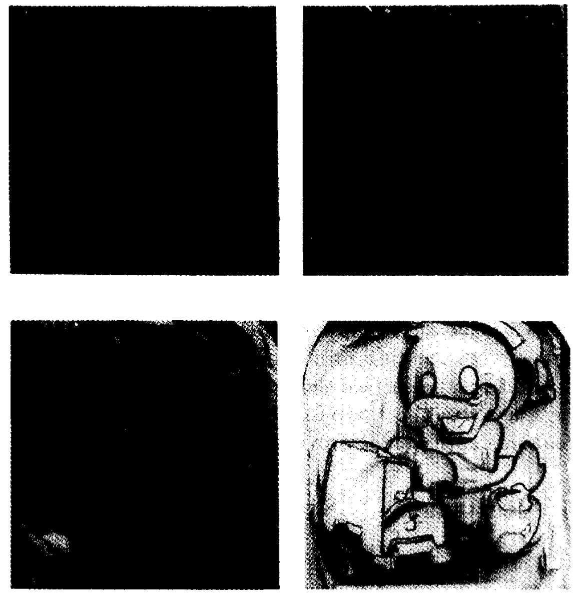

Figure 4: Shaded depth map from stereo, shaded filtered depth map, and the final result illuminated from $(1,1,1)/\sqrt(3)$ and $(0,0,1)$

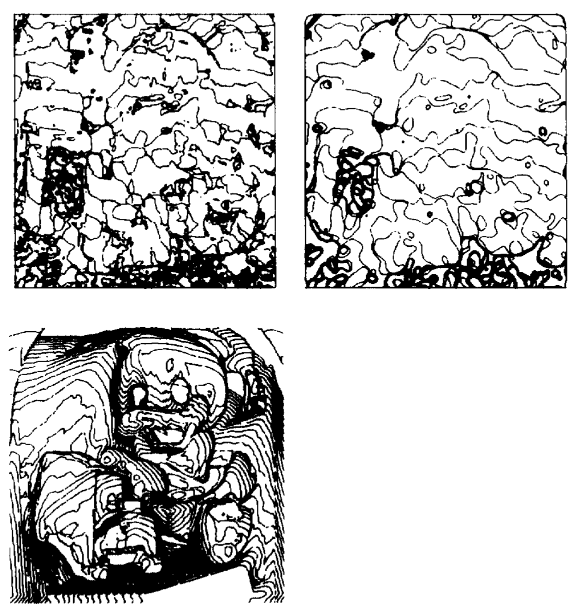

Figure 5: Level curves of the stereo result, level curves of the filtered stereo result and the level curves of the final surface.